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Traces of Time: Decoding History from Staircase Wear

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ABSTRACT
The analysis of residual information on stairs that have been worn over time is of great significance for understanding historical contexts. This study combines research on human walking patterns with daily observations of various walking behaviors to develop models for walking patterns and step-landing probabilities. Using an approximation approach, we establish Crowd Flow Pattern Recognition Model, aiming to infer how people used the stairs several centuries ago. Furthermore, by applying Archard's model and accounting for factors such as humidity, temperature, crowd flow, and material hardness, we establish Wear Volume-Time Relationship Model, which plays a crucial role in predicting the usage lifespan of worn stairs.

KEYWORDS
Wear estimation; Archard's model; Possibility model; Monte carlo method; Critical time search method

1 Introduction

People have passed away, but their footprints stay. By analyzing worn stones, we can get lots of useful information. Both stones and wood are common materials used for constructing steps. Over time, these building materials may experience uneven wear, which can provide insights into the history and some other information of the structures or offer guidance on complex issues.

1.1 Contributions

We have innovatively proposed that the direction, number, and frequency of foot traffic influence the distribution of wear depth on the surface of worn stairs. In practical applications, by analyzing the weight coefficient of cases of different using patterns we proposed, we can deduce how the worn stairs were used in a given year. Additionally, we have creatively integrated temperature, humidity, and crowd flow with Archard's wear model, establishing the relationship between stair wear and usage lifespan, which provides valuable reference for archaeologists in analyzing the age of stairs.

1.2 Organization of the paper

In the section 2, we gave the prior knowledge of our models and some main assumptions we used. In the section 3, we first outlined the approach and process for constructing the Crowd Flow Pattern Recognition Model, then we analyzed specific cases in details in section 3.2, 3.3 and 3.4. In the section 4, we show the process for constructing Wear Volume-Time Relationship Model. The last section is the conclusion.

1.3 Notation

It should be mentioned that when analyzing different situations, we would process matrix differently. Therefore, the dimension of $\delta^{(i)}$ varies in each case.

Table 1

Symbol	Meaning
k	the wear coefficient of Archard's model
H	the hardness of the material
A	area of the foot sole
D	matrix which represents the worn condition of the stair.
$\delta^{(0)}$	matrix generated from original matrix which represents the worn condition of a real stair
$\delta^{(1)}$	the processed matrix derived from simulation where only one person walk each time in different directions
$\delta^{(2)}$	the processed matrix derived from simulation where only one person walk each time upstairs
$\delta^{(3)}$	the processed matrix derived from simulation where only one person walk each time downstairs
$\delta^{(4)}$	the processed matrix derived from simulation where two persons walk simultaneously each time in different directions
$\delta^{(5)}$	the processed matrix derived from simulation where n persons walk simultaneously each time in different directions
δ	matrix best approximating $\delta^{(0)}$
c_{i_n}	weight coefficient, $\delta = \sum_{n=i_1}^{i_n} c_{i_n} \delta^{i_n}$

2 Prior knowledge and some assumptions

2.1 Archard model

The Archard ' s model, a classic model to estimate the wear of material is defined as below.

$$V = k\frac{W}{H} = k\frac{F_nL}{H}.$$

k is the wear coefficient which is determined by temperature, humidity, time and the material properties. Based on various studies, the wear coefficient of the stone typically ranges from 10^{-5} to 10^{-3}, while for the wood, it ranges from 10^{-6} to 10^{-3}. In this section, we suppose the material of the stair is stone. When the stair is made of wood, the model is the same. H is used to quantify the hardness of material. According to the data, H of the stone is approximately $10^7$to 10^8 Pa, while that of the wood usually range from 10^6 to 10^7Pa. F_n denotes the pressure on the surface of the staircase. To simplify the following calculations, we consider the average weight of a person, around 65 kg, as the value of F_n. Addition- ally, the average friction distance L between people ' s step and the ground is found to be 10^{-3}m.

With all the information above, we can calculate the volume of the staircase worn down by one step. Given an average plantar area, typically $250*10^{-4}m^2$, the average depth worn by one step can be determined.

2.2 Assumptions

In order to streamline our model we have made several key assumptions.

• When the steps are relatively short and only one person is walking on the staircase, the majority tend to walk in the middle of the steps, while a smaller proportion pre- fer walking along the edges. This preference follows a certain normal distribution.

• The wear on the stairs is primarily caused by two factors: human foot traffic and environmental damage.

• The depressions created by previous footsteps do not influence the choice of foot placement for subsequent users.

• When there are fewer people on the staircase and it is more spacious, individuals prefer walking in the middle.

• When the staircase is crowded with more people, the distribution of individuals becomes more dispersed, and it is more likely for multiple people to walk side by side.

• A single step can accommodate a maximum of n people at the same time.

3 Theoretical analysis of the crowd flow pattern recognition model

3.1 Overview of the model

3.1.1 Estimation of walking pattern and step-landing position

Based on relevant literature and empirical observations, we posit that the greater the frequency of wear at a specific location on a step, the deeper the wear depth at that location. Furthermore, this relationship is assumed to be positively linear. Consequently, the distribution of wear depth on the step can be used to infer the probability distribution of footfall at each point.

Leveraging the law of large numbers and the approximation of the binomial distribution to the normal distribution, it can be assumed that each footfall occurring at a speci fi c point is an independent and identically distributed trial, following a binomial distribution.

$$f(x,y) = \frac{d(x,y)}{\int_S d(x,y)\,dS},$$

$$N = \frac{\Delta V}{d_0 A}.$$

Where:

• $f(x,y)$ is the probability distribution function of a single step being trodden on the staircase plane;

• $d(x,y)$ represents the depth of the pit at point (x,y) on the staircase plane;

• S represents the staircase plane;

• $d_0 = \frac{kW}{HA}$ represents the average depth of a single step and A represents the area of the foot;

• ΔV represents the volume of the staircase step that has been worn away;

• N represents the total number of people passing through the step.

3.1.2 Stairs wearing simulation

In specific calculation, we assume the size pf a stair is $1*0.4*0.3m^2$ and after meshing the staircase plane, we use a matrix $\delta \in R^{90*30}$ to represent the wear condition of the upper surface. (Notice: δ in different sections has different meanings.) $\delta_{i,j}$ represents the wear depth at position $\left(x_i,y_j\right)$. A_0 is the area of a grid square.

$$\delta_{i,j} = Nf\left(x_i,y_j\right)A_0 d_0.$$

Using the Monte Carlo method, we sample the stepping positions on the staircase to calculate the wear depth of that certain area. It should be noticed that the edge of the stair would be worn faster than the inner part due to some natural factors and different directions of applied force. Therefore, we improve the Archard ' s model when calculate the worn depth of specific point $\delta\left(i,j\right)$ for one step: (r means the length of the row)

$$h = k\frac{F_n L}{HA}e^{\frac{i-1}{r-1}}.$$

3.1.3 Estimating traffic patterns with the idea of approximation and linear programming

Suppose $\delta^{(0)}$ processed from the worn matrix of a real stair, and $\delta^{\left(i_1\right)}\cdot\cdot\delta^{\left(i_n\right)}$ are the matrixes , which represent the wearing simulations in extreme situations. Now, we see $\delta^{\left(i_1\right)}\cdot\cdot\delta^{\left(i_n\right)}$ as a set of vectors and they form a subspace M of R^{m*n}. Our goal is to find the vector δ best approximating $\delta^{(0)}$ in the subspace, which can also be called search scope there. Based on the theory of best least-squares approximation, we get:

$$\left(\delta - \delta^{(0)},\delta^{\left(i_1\right)}\right) = 0,$$

$$\cdots$$

$$\left(\delta - \delta^{(0)},\delta^{\left(i_n\right)}\right) = 0.$$

and

$$\delta = c_{i_1}\delta^{\left(i_1\right)} + \cdots + c_{i_n}\delta^{\left(i_n\right)}.$$

After solving the system of equations above, we can obtain coefficients $c_{i_1}\cdot\cdot c_{\left(i_n\right)}$. Analyzing the coefficients, we get something about people ' s traffic patterns. Specific analyzing is shown below.

To get more accurate results, we could impose some constraints to limit search scope. For example, let all coefficient $c_i \geq 0\left(i = 1 \cdots n\right)$. Then we use methods for solving linear programming problems to obtain more precise results.

3.2 Recognition of directions

Firstly, we assume that a person is more likely to step on the middle of a stair when walking alone. Establishing a Cartesian coordinate system on the stair plane as below, we assume that the position coordinates of a person ' s foot in the x-axis direction and the y-axis direction are independently and identically distributed, both following a normal distribution. By performing numerical experiments, the following value of mean and variance is appropriate. (unit: cm)

$$x \sim N\left(45,20\right),$$

$$y \sim N\left(25,5\right).$$

From research, when people walk, the forefoot bears greater load and pressure compared to the heel. We assume that the load distribution between the forefoot and the heel follows a 7:3 ratio. Then, we use Monte Carlo method to simulate the wear of stair in two situations: people all walking upstairs and people all walking downstairs. The result is shown below.

Comparing the results, It ' s apparent that the difference in the impact of depth on the wear between walking upstairs and downstairs is primarily reflected in the row direction of the simulation matrix. Therefore, the method to process the wear matrix is to sum up each row of the matrix to form vectors $\delta^{(1)},\delta^{(2)}$, which represent vectors for people walking upstairs and downstairs, and $\delta^{(0)}$, which represents the vector for the real situation.

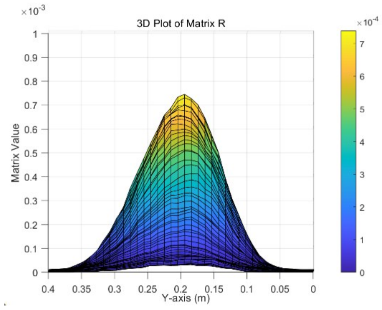

Figure 1 Upstairs/the trend on the row

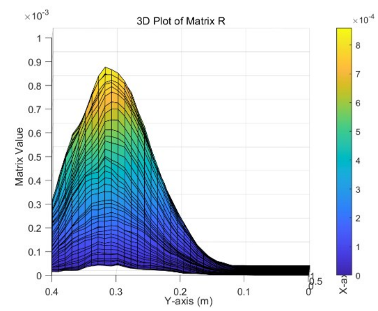

Figure 2 Downstairs/the trend on the row

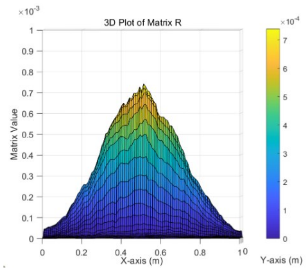

Figure 3 Upstairs/the trend on the column

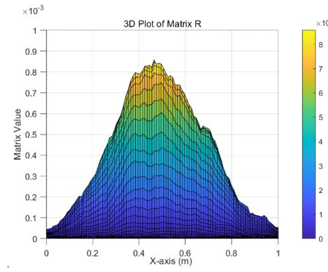

Figure 4 Downstairs/the trend on the column

Solving equations listed in the 3.1.3, we can gain the coefficients c_1 and c_2.

$$\left(\delta^0-\delta,\delta^{(1)}\right)=0,$$

$$\left(\delta^{(0)}-\delta,\delta^{(2)}\right)=0,$$

$$\delta=c_1\delta^{(1)}+\delta^{(2)}.$$

Let

$$C=\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

and

$$F=\left(\delta^{(1)}\delta^{(2)}\right).$$

The equations turn into:

$$\begin{pmatrix} \delta^{(1)}*\delta^{(0)} \\ \delta^{(2)}*\delta^{(0)} \end{pmatrix}=\begin{pmatrix} \delta^{(1)}*F \\ \delta^{(2)}*F \end{pmatrix}C.$$

Then we can calculate the ratio of people walking upstairs to those walking downstairs using c_1 and c_2 approximately.

$$p_1=\frac{c_1}{c_1+c_2},$$

$$p_2=\frac{c_2}{c_1+c_2}.$$

We can figure out that about p_1 of people walk upstairs and about p_2 of people walk downstairs.

3.3 Recognition of the number of people using stairs simultaneously

In this section, we only talk about people walking alone or two people walking at the same time. The discussion of a number of people stepping on one stair simultaneously is in next section.

We still assume that a person is more likely to step on the middle of a stair when walking alone. However, when two people walk on one stair simultaneously, we assume that people prefer to walk almost symmetrically on both sides of the stairs. $\left(x_1,y\right)$ represents the step position of the first person and $\left(x_2,y\right)$ represents the step position of the second person. Like subsection above, we still assume that the position coordinates of a person ' s foot in the x-axis direcion and the y-axis direction are independently and identically distributed, both following a normal distribution. Furthermore, to simplify calculation, we assume that x_1 and x_2 are independently and identically distributed, following a normal distribution too. By performing numerical experiments, the following values of mean and variance is appropriate: (unit: cm)

$$x_1\sim N\left(30,15\right),$$

$$x_2\sim N\left(70,15\right),$$

$$y\sim N\left(25,5\right).$$

The simulation results are shown below.

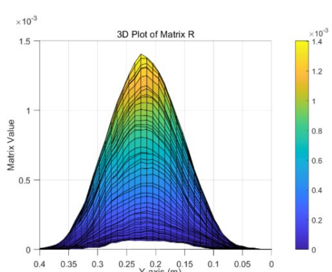

Figure 5 People all walk alone/the trend on the row

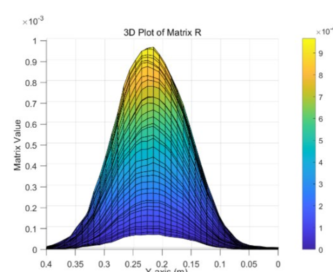

Figure 6 Two people walk simultaneously/the trend on the row

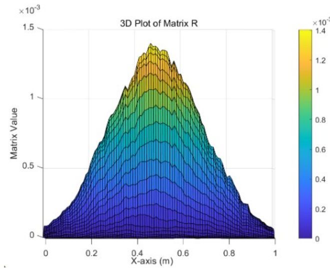

Figure 7 People all walk alone/the trend on the column

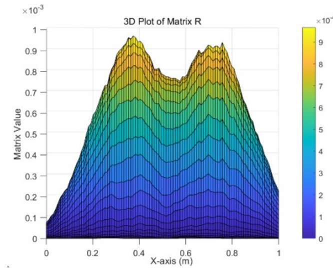

Figure 8 Two people walk simultaneously/the trend on the column

From the figures we can find that the depth of wear on column of two simulation matrix has huge difference. Therefore, we add up each column of the matrix to form vectors $\delta^{(3)},\delta^{(4)}$, which represent vectors for people walking alone or two people walking at the same time. Similar to subsection above, after solving equations in 3.1.3, we gain the coefficients c_3 and c_4.

$$p_3=\frac{c_3}{c_3+c_4},$$

$$p_4=\frac{c_4}{c_3+c_4}.$$

We can figure out that the percentage of people walk alone is around p_3 and the percentage of two people walk simultaneously is p_4.

3.4 Recognition of intensive short-term use and chronic low-level use

We will provide a certain level of prediction regarding the daily usage of the staircase. Specifically, given the fixed number of users in a day N and the observed wear distribution on the staircase, we aim to determine whether the staircase is more likely to experience short-term heavy traffic or long-term light traffic. Based on the relevant information online and empirical observations, we conclude that:

• Within a single day, changes in temperature and humidity are insufficient to significantly affect the wear coefficient k and hardness of the staircase material H.

• The effects of high-frequency impacts and low-frequency impacts on the wear of the staircase differ. Under high-frequency impacts, the material's hardness H de- creases slightly at the moment of impact.

Based on the above analysis, the position where a person walks on the stairs can be considered as a two-dimensional independent normal random vector. The probability density function is:

$$f_1=\frac{1}{2\pi\sigma_x\sigma_y}e^{-\left[\frac{(x-x_c)^2}{2\sigma_x^2}+\frac{(y-y_c)^2}{2\sigma_y^2}\right]},$$

where (x_c,y_c) represents the center of the staircase and (σ_x,σ_y) represents the variance. The case where a small number of people passed through the stairs over a long period is similar to 3.2.2. For a large number of people passing over the steps within a short period, if q people stand side by side on the steps, the wear patterns are likely to exhibit a multi-peak distribution along the x-axis, showing periodic variations. The period length is approximately $\frac{l_x}{q}$, where l_x represents the length of the staircase. To better describe the regular distribution along the x-axis under such side-by-side scenarios, trigonometric functions are effective. Meanwhile, the distribution along the y-axis continues to follow a Gaussian distribution. Therefore, the overall distribution function can be expressed as:

$$f_2=\frac{1}{\sqrt{2\pi}\,\sigma_y}e^{\frac{(y-y_c)^2}{2\sigma_y^2}}\left[1+\cos\left(\frac{2q\pi}{l_x}x\right)\right],\ \text{for } 1\le q\le n.$$

The results are shown below.

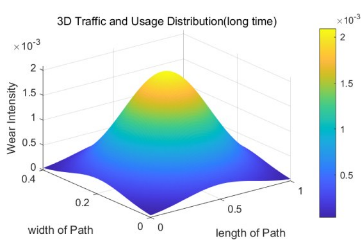

Figure 9 Wear depth of long time and fewer people

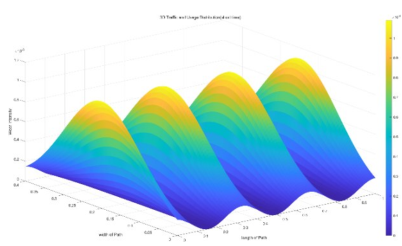

Figure 10 Wear depth of short time and more people

We gain the matrixes $\delta^{(3)}$ and $\delta^{(5)}$. Similarly, solve the equations in 3.1.3, we can find out the probabilities of the crowd pattern involves a large number of people passing through in a short period of time or not.

4 Theoretical analysis of the wear volume-time relationship model

4.1 Estimating the age

Given an approximate estimate of the stairwell's age, its usage patterns, and an estimate of daily life activities within the structure, we essentially have a rough estimate of the stairwell's age and the approximate number of people using the stairs each year.

We still use the Archard model to calculate the wear depth and wear volume of the steps. Considering the influence of environment and time on the material coefficient in the Archard model, we will obtain the relationship between wear volume and time. By substituting the actually measured wear volume into the wear volume-time relationship, we can solve for time, that is, the age of the stairwell.

Below, we provide a detailed description and derivation of this estimation method, and proceed to solve it.

Assume the approximate age of the staircase is T, and the cumulative number of people using the staircase in the year t is $n(t)$. We use years as the unit of examination, then $t = 1,2,\cdots,T$.

4.2 Wear volume-time relationship model

As we already know, the core formula of the Archard model is:

$$V = k\frac{FL}{H}.$$

By dividing by the contact area A, we can derive the formula for average wear depth for a single person:

$$d_0 = k\frac{FL}{HA}.$$

It can be seen that the wear depth per person is primarily determined by the material properties, and thus the cumulative wear depth is determined by the material and the cumulative number of users.

We define the material coefficient as a comprehensive coefficient that integrates material properties, and thus the wear depth of year t can be expressed as:

$$d(t) = K_0(t)\,n(t).$$

Where $K_0(t)$ is the material coefficient, $n(t)$ is the number of users in the year t. Both K_0 and n are time-dependent.

Next, we provide the calculations for $K_0(t)$ and $n(t)$. Considering the influence of environmental factors (temperature and humidity), we set the baseline decay rate as ϕ_0. And we adjust the baseline decay rate based on these factors to obtain the environmental decay coefficient ϕ:

$$\phi = \phi_0\left[1 + \alpha(\xi - \xi_0) + \beta(\eta - \eta_0)\right],$$

where α is the temperature influence coefficient, ξ is the temperature, β is the humidity influence coefficient, and η is the humidity.

(Explanation: The linear form of the environmental decay coefficient correction formula is set here because the linear form can greatly simplify the complexity of the model and improve computational efficiency. In practical applications, the linear model usually can approximate the relationship between material properties and environmental factors well.)

For K is affected by environmental factors and decaying over time, we set:

$$K_0(t) = \phi\gamma_1\left(1 - e^{-\gamma_2 t}\right).$$

From the Archard model mentioned above, it can be seen that $\phi_0 = \frac{k}{H}$, and substituting the data from the previous context gives approximately $2*10^{-11}\mathrm{Pa}^{-1}$, $\gamma_1 = \frac{F\cdot s}{A}$, and substituting the data from the previous context gives approximately 280N/m.

Considering the actual situation, historical buildings attract tourists immediately after their construction, but over time, the number of visitors gradually decreases due to the buildings' wear and the need for preservation. That is, $n(t)$ initially increases with time t and then decreases with time t. Therefore, we set $n(t)$ as a piecewise function of time t. In this problem, we will analyze using the following piecewise function for $n(t)$ as an example.

$$n(t) = \begin{cases} 20000 + \dfrac{200 + n_0\left[\dfrac{\pi}{2} - \arctan(50\omega)\right] - 20000}{50}t & t \leqslant 50 \\ 200 + n_0\left[\dfrac{\pi}{2} - \arctan(\omega t)\right] & t \geqslant 50 \end{cases}$$

where n_0 is the average number of people using the stairs per year, set to 20000.

Finally, we calculate the cumulative wear depth and wear volume. Based on the analysis from the previous context, we have obtained:

$$d(t)=K_0(t)n(t).$$

And the wear volume in year t is:

$$V(t)=\sum_{\tau=0}^{t} d(\tau).$$

By substituting the measured wear volume $V_{measured}$ into the wear volume-time relationship, we obtain the equation:

$$V_{measured}=\sum_{\tau=0}^{t} d(\tau).$$

Solving for t gives us desired result.

4.3 Critical time search method

To solve the model, we consider using the Critical Time Search Method. By searching step by step, we find the critical time point that meets specific conditions to solve the equation. That is, in the time series, find the first time point t that satisfies:

$$V(t)\le V_{measured}\le V(t+1).$$

The t that meets the condition is the integer solution of the equation.

Algorithm 1 Critical Time Search Method

```
Initialize predicted_age to NaN
for t = 1 to length(time_steps) −1 do
    if wear_vol(t) ≤ measured_wear_vol and wear_vol(t+1) > measured_wear_vol then
        predicted_age = time_steps(t)
        break
    end if
end for
if isnan(predicted_age) then
    display "No valid step age found."
else
    display "Predicted step age is: " predicted_age " years"
end if
```

5 Conclusion

In this study, we investigated the relationship between stair wear patterns, human walking behavior, and the temporal evolution of material degradation. By introducing a Crowd Flow Pattern Recognition Model and a Wear Volume – Time Relationship Model, we provided a systematic framework for interpreting the residual information preserved on worn stairs. Our approach combined probability-based walking and step-landing models with Archard ' s wear model, integrating environmental and material factors such as humidity, temperature, crowd flow, and hardness.

The results demonstrate that variations in walking direction, stepping position, and usage frequency significantly shape the depth distribution of stair wear, enabling us to infer historical patterns of stair usage. Furthermore, the integration of wear dynamics with environmental parameters allowed us to establish a robust relationship between wear volume and stair age, offering archaeologists a practical tool for dating architectural remains.

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